Technical Comments

Comment on "The Lift Force Due to von Kármán's Vortex Wake"

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THERE have been a number of papers relating the fluctuating lift force on a bluff body to the vorticity shed in a periodic von Kármán wake.[1-4] Sallet[5] erroneously implies that they can only be applied to circular cylinders, and that his analysis has a more general application. In fact, as observed by Birkhoff and Zarantonello,[6] for example, any such analysis involves three unknowns: Γ, the circulation in each vortex; h, the spacing of the two (assumed parallel) vortex rows; and l, the space between each vortex in a row.

Alternatively, the translational velocity u_s of the vortex street with respect to the fluid (u_o) can be treated as an unknown, enabling l to be found from a relationship such as[7]

$$u_s = (\Gamma/2l)\tanh(\pi h/l)$$

$$\tanh \pi h/l = 1/(2)^{1/2} \qquad \text{(for stability)}$$

So long as these three parameters are known for a given cylinder shape, the lift force can in principle be calculated by any of the methods in the literature. Sallet's main contribution would seem to be in pointing out that one of the unknowns can be obtained from the drag equation. It is also possible that his relationship between lift and circulation is more accurate than previous formulations; the writer has made no attempt to check this aspect of his paper.

The equations for the maximum lift coefficient obtained by the investigators listed[1-5] are as follows:

Ruedy[1] (1935) $$C_L = \pm(1/2)(\Gamma/u_o d)[1 - (u_s/u_o)] \quad (1)$$

Steinman[2] (1947) $$C_L = \pm(1/2)(\Gamma/u_o d) \quad (2)$$

(Steinman was apparently not aware of Ruedy's work, and used the steady-state lift equation $L = \rho\Gamma u_o$.)

Chen[3] (1972) $$C_L = \pm 2(\Gamma/u_o d)(u_s/u_o) \quad (3)$$

Chaplin/Chen[4] (1972) $$C_L = \pm(\Gamma/u_o d)(1 - u_s/u_o) \quad (4)$$

Chaplin[4] (1972) $$C_L = \pm(\Gamma/u_o d)(1 - u_s/u_o)(1 - \Omega) \quad (5)$$

[To obtain Eq. (5), Chaplin replaces the first four inviscid vortices with Hamel-Oseen vortices, to account for viscosity. Ω is a term reflecting the effects of viscosity, and depends on the viscous vortex core radius.]

Sallet[5] (1973) $$C_L = \pm(1/2)(\Gamma/u_o d)[1 - 3(u_s/u_o)] \quad (6)$$

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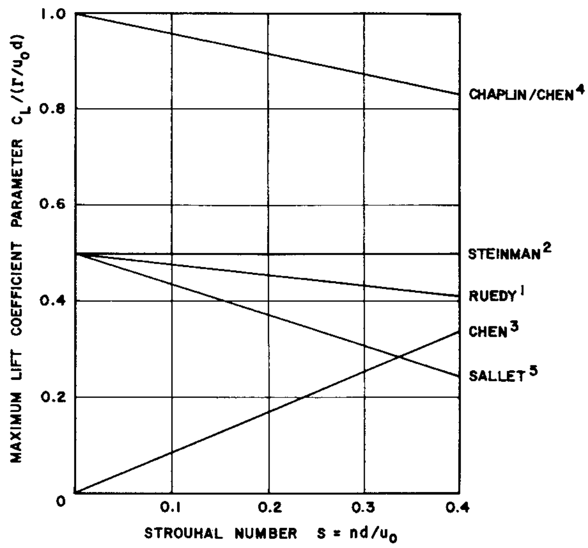

Fig. 1 Various expressions for maximum lift coefficient as a function of circulation Γ and Strouhal number, assuming inviscid wake vortices.

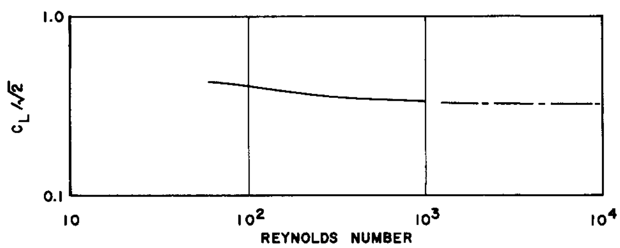

Fig. 2 Variation of lift coefficient of a circular cylinder with Reynolds number, according to Sallet's theory.

It is of some interest to compare these results, using Shaw's[8] correlation, which gives, for a circular cylinder

$$u_s/u_o = (4/3\pi)S \quad (7)$$

where S is the Strouhal number nd/u_o. As can be seen from Fig. 1, there is considerable variation among the various authors.

Like most others, Sallet's equations give a lift coefficient which is almost invarient with Reynolds number, as indicated in Fig. 2. (To compute this, the Strouhal number (S) and drag coefficient (C_D) were obtained from Ref. 18 for $R < 10^3$. Above $R = 10^3$, we assume $S = 0.2$, $C_D = 1.15$, as being approximately correct up to transition.)

Comparing this result with the available experimental data (Fig. 3),† we see that Sallet's result is too low, by a factor of 2 or 3, in the high subcritical Reynolds number range. At low Reynolds numbers, it is apparently too high, by one or more orders of magnitude, when compared with the data of Gerrard. But this latter discrepancy bears some further discussion.

It seems quite clear from Gerrard's data—some hundreds of direct static pressure measurements on the surfaces of cylinders—that C_L decreases with decreasing R, below $R = 2 \times 10^4$. As Gerrard says, "... in the range 4000 $< R <$ 20,000, the experiments indicate a power law relation between lift coefficient and Reynolds number, viz,

†References 9–14 were not cited in Sallet's paper.

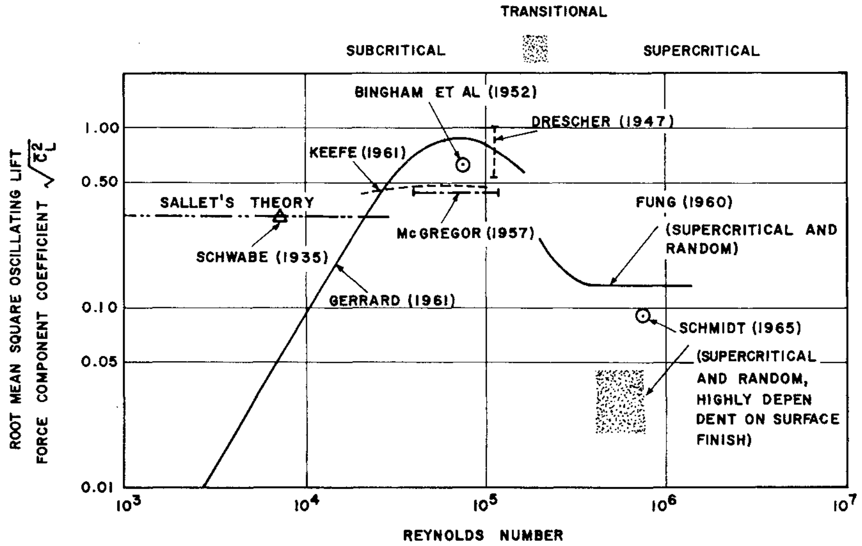

Fig. 3 Magnitude of the lift force coefficient on a rigid circular cylinder, as a function of Reynolds number.

$$(\bar{C}_L{}^2)^{1/2} \propto R^{1.7}$$

. . . ." Moreover, in a later paper[21] Gerrard confirms this result theoretically, by approximate numerical solutions for the viscous flow over a cylinder. As against this rather large body of evidence, we have two isolated data points, which do give much higher values of C_L at low R, and tend to agree with Sallet's theory.

The first of these is due to Schwabe,[15] cited by Sallet in one of his two comparisons with experiment. Schwabe found $C_L = 0.45$ at $R = 735$, by measuring the fluid velocity from photographs of an impulsively started cylinder. One feels that there are enough unknowns in this early work (nonsteady state, the possibility of cylinder vibration) for it to be disregarded in comparison with Gerrard's data.

The second data point which disagrees with Gerrard was deduced by Phillips[19] from Kovasznay's[20] hot wire measurements in the wake of cylinders, giving $C_L \simeq 0.75$ for $40 < R < 180$. Again, this was a much less direct approach than the pressure measurements of Gerrard.

Sallet also compared his theoretical predictions with data from Fung,[17] but the latter's experiments were in the supercritical range, and his paper presents data to show that no coherent vortex street was present.‡ The good agreement therefore must clearly be fortuitous. Also, note from Fig. 3 that, depending on surface finish, Schmidt[16] records C_L values almost an order of magnitude lower than Fung's.

If Gerrard's data is more or less correct, it would follow that the circulation of the discrete vortices in the wake becomes relatively less as the Reynolds number decreases, and more of the drag is attributable to other mechanisms. In other words, the total drag can be expressed as

$$C_D = \Delta C_{D_{\text{skin friction}}} + \Delta C_{D_{\text{von Karman}}} + \Delta C_{D_{\text{other}}}$$

We already know that

$$\text{For } R < 100 \qquad \Delta C_{D_{\text{von Karman}}} = 0$$

$$\text{For } R > 2 \times 10^5 \quad \Delta C_{D_{\text{von Karman}}} = 0 \quad \Delta C_{D_{\text{skin friction}}} \simeq 0$$

so that

$$\Delta C_{D_{\text{other}}} \simeq C_D$$

Why then, in the intermediate range, should we suppose that all the drag energy appears in the discrete vortex street? The writer would like to suggest that while a large portion does, around $R = 10^5$, this proportion becomes smaller as R decreases, and, of course, vanishes around $R = 100$. If this turns out to be correct, then the several attempts which have been made to calculate lift from grossly over-simplified models will face a new difficulty, that of defining the discrete vortex circulation.

In conclusion, a rather different difficulty is experienced with Sallet's statement that his theory is "valid for any bluff body in cross flow, so long as the stated assumptions are satisfied." The difficulty is that a thin flat plate, normal to the flow, undoubtedly develops a vortex street, but is incapable of experiencing a lift force.

‡To quote Fung, "The most important feature of the forces induced on a circular cylinder by vortex shedding in a supercritical Reynolds number range, is their randomness."

References

1Ruedy, R., "Vibrations of Power Lines in a Steady Wind," *Canadian Journal of Research,* Vol. 13, Sec. A, No. 4, Oct. 1935, pp. 82–92.

2Steinman, D. B., "Problems of Aerodynamic and Hydrodynamic Stability," *Proceedings of the 3rd Hydraulics Conference,* Bulletin 31, Iowa State Univ. of Science and Technology, Ames, Iowa, 1947, pp. 136–163.

3Chen, Y. N., "Fluctuating Lift Forces of the Kármán Vortex Streets on Single Circular Cylinders and in Tube Bundles. Part 2—Lift Forces of Single Cylinders," *Journal of Engineering for Industry,* May 1972, pp. 613–618.

4Chaplin, J. R., Discussion, *Journal of Engineering for Industry,* May 1972, pp. 619–622.

5Sallet, D. W., "The Lift Force Due to von Kármán's Vortex Wake," *Journal of Hydronautics,* Vol. 7, No. 4, Oct. 1973, pp. 161–165.

6Birkhoff, G. and Zarantonello, E. H., *Jets, Wakes and Cavities,* Academic Press, New York, 1957.

7von Kármán, Th. and Rubach, H., "Uber den Mechanismus des Flussigkeits und Luftwiderstandes," *Physikalische Zeitschrift,* No. 2, Jan. 1912, pp. 49–59.

8Shaw, R. A., "An Explanation of Vortex Shedding on the Basis of Pulses Travelling at the Speed of Sound," Rept. 18455, 1956, Aeronautical Research Council, London, England.

9Gerrard, J. H., "An Experimental Investigation of the Oscillating Lift and Drag of a Circular Cylinder Shedding Turbulent Vortices," *Journal of Fluid Mechanics,* 1961, pp. 245–256.

10Bingham, H. H., Weimer, D. K., and Griffith, W., "The Cylinder and Semicylinder in Subsonic Flow," TR II-13, 1952, Princeton Univ., Princeton, N.J.

11Drescher, H., "Model Testing Techniques. II.6: Measurement of Unsteady Pressure," AVA Monographs D2, Rept. 11,391, 1947, Aeronautical Research Council, London, England (translation).

[12]McGregor, D. M., "An Experimental Investigation of the Oscillating Pressures on a Circular Cylinder in a Fluid Stream," TN 14, 1957, University of Toronto Institute of Aerophysics, Toronto, Ontario, Canada.

[13]Humphreys, J. S., "On a Circular Cylinder in a Steady Wind at Transition Reynolds Numbers," *Journal of Fluid Mechanics,* Vol. 9, 1960, p. 603.

[14]Keefe, R. T., "An Investigation of the Fluctuating Forces Acting on a Stationary Circular Cylinder in a Subsonic Stream, and of the Associated Sound Field," UTIA Rept. 76, AFOSR 2147, Sept. 1961, University of Toronto Institute of Aerophysics, Toronto, Ontario, Canada.

[15]Schwabe, M., "Uber Druckermittlung in der nichtstationaren e ebenen Stromung," *Ingenieur-Archiv,* Vol. 6, Feb. 1935, pp. 34–50.

[16]Schmidt, L. V., "Measurements of Fluctuating Air Loads on a Circular Cylinder," *Journal of Aircraft,* Vol. 2, No. 1, Jan. 1965, pp. 49–55.

[17]Fung, Y. C., "Fluctuating Lift and Drag Acting on a Cylinder in a Flow at Supercritical Reynolds Numbers," *Journal of Aerospace Science,* Vol. 27, 1960, p. 801.

[18]Schlichting, H., *Boundary Layer Theory,* McGraw-Hill, New York, 1968, p. 32, Fig. 2.9.

[19]Phillips, O. M., "The Intensity of Aerolian Tones," *Journal of Fluid Mechanics,* Vol. 1, 1956, pp. 607–624.

[20]Kovasznay, L. S. G., "Hot Wire Investigation of the Wake Behind Cylinders at Low Reynolds Numbers," *Proceedings of the Royal Society London,* Ser. A, Vol. 198, 1949, p. 174.

[21]Gerrard, J. H., "Numerical Computation of the Magnitude and Frequency of the Lift on a Circular Cylinder," *Philosophical Transactions of the Royal Society London,* Ser. A, Vol. 261, No. 1118, Jan. 1967.

Reply by Author to P. R. Payne

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I do not agree with Payne's conclusion for two reasons:

1) The von Kármán vortex wake I chose as a model for the real wake yields a prediction equation for the coefficient of lift [$C_L = C_L(S, C_D)$] which is very sensitive to differences in the Strouhal number S and in the coefficient of drag C_D. Therefore, a valid comparison between the theoretical results and experimental results is only possible if, during the same experiments, the lift as well as the drag and the Strouhal number were measured simultaneously. Figure 1 shows how slight changes of the coefficient drag C_D and the Strouhal number S result in large changes of the predicted lift coefficient C_L.

2) Various experiments[1-10,12-15] gave vastly different results for the coefficient of lift (see Fig. 1). The statement that a theory does not correctly predict a particular set of experimental data could in this case be rephrased to say that the experimental results in question are not supported by the theory.

I believe that the dispersion of the different experimental results is probably due to the fact that it is very difficult to keep a cylinder in cross flow perfectly at rest. The fact that the lift force can be drastically reduced as well as drastically increased due to the vibration of the cylinder is well documented. The inclusion of the effects of vibration was discussed on p. 164 of my paper[11] in a preliminary fashion. It is seen that my theory correctly predicts the trends of the increase or the decrease of the lift when the cylinder vibrates with a certain frequency.

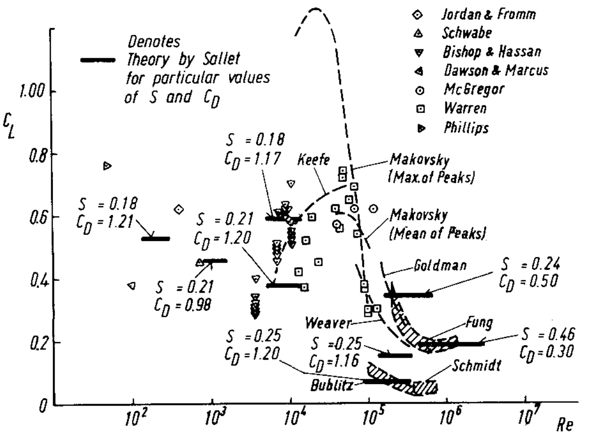

Fig. 1 The coefficient of lift C_L as a function of the Reynolds number.

References

[1] Bishop, R. E. D. and Hassan, A. Y., "The Lift and Drag Forces on a Circular Cylinder in a Flowing Fluid," *Proceedings of the Royal Society London,* Ser. A, Vol. 277, 1963, pp. 32–50.

[2] Bublitz, P., "Messungen der Drücke und Kräfte am ebenen, querangeströmten Kreiszylinder, Teil I, Untersuchungen am ruhenden Kreiszylinder," DFVLR—Aerodynamische Versuchsanstalt Göttingen, 71 J 11, 1971, p. 44.

[3] Dawson, C. and Marcus. M., "DMC-A Computer Code to Simulate Viscous Flow About Arbitrarily Shaped Bodies," *Proceedings of the 1970 Heat Transfer and Fluid Mechanics Institute,* Stanford University Press, Stanford, 1970, pp. 323–338.

[4] Fung, Y. C., "Fluctuating Lift and Drag Acting on a Cylinder in a Flow at Supercritical Reynolds Numbers," *Journal of Aerospace Science,* Vol. 27, No. 11, Nov. 1960, pp. 801–814.

[5] Goldman, R. L., "Kármán Vortex Forces on the Vanguard Rocket," *The Shock and Vibration Bulletin,* No. 26, Pt. 2, Dec. 1958, pp. 171–179.

[6] Jordan, S. K. and Fromm, J. E., "Dynamic Interaction Between a Circle Moving at Terminal Velocity and the Surrounding Fluid Medium," AIAA Paper 72-111, San Diego, Calif., 1972, pp. 1–9.

[7] Keefe, R. T., "Investigation of the Fluctuating Forces Acting on a Stationary Circular Cylinder in a Subsonic Stream and of the Associated Sound Field," *Journal of the Acoustical Society of America,* Vol. 34, No. 11, Nov. 1962, pp. 1711–1714.

[8] Macovsky, M. S., "Vortex Induced Vibration Studies," Rept. 1190, July 1958, David Taylor Model Basin, Washington, D.C. (as given by Ref. 7, p. 1712, Fig. 2).

[9] McGregor, D. M., "An Experimental Investigation of the Oscillating Pressures on a Circular Cylinder in a Fluid Stream," TN June 1957, University of Toronto Institute of Aerophysics, Toronto, Ontario, Canada.

[10] Phillips, O. M., "The Intensity of Aeolian Tones," *Journal of Fluid Mechanics,* Vol. 1, Pt. 6, Dec. 1956, pp. 607–624.

[11] Sallet, D. W., "The Lift Force Due to von Kármán's Vortex Wake," *Journal of Hydronautics,* Vol. 7, No. 4, Oct. 1973. pp. 161–165.

[12] Schmidt, L. V., "Measurements of Fluctuating Air Loads on a Circular Cylinder," *Journal of Aircraft,* Vol. 2, No. 1, 1965, pp. 49–55.

[13] Schwabe, M., "Ueber Druckermittlung in der nichtstationären ebenen Strömung," *Ingenieur-Archiv,* Vol. 6, Feb. 1935, pp. 34–50.

[14] Warren, W. F., "An Experimental Investigation of Fluid Forces of an Oscillating Cylinder," Ph.D. dissertation, 1962, University of Maryland, College Park, Md.

[15] Weaver, W., Jr., "Wind-Induced Vibrations in Antenna Members," *Proceedings of the ASCE,* Vol. 87, 1961, EM1, p. 141 (as given by M. Pflügl, "Eine Versuchseinrichtung zur Messung der aerodynamischen Kräfte an schwingenden Körpern und deren Anwendung auf die Bestimmung der aerodynamischen Querkraft an einem schwingenden Kreiszylinder," Ph.D. dissertation, Fakultät für Maschinenwesen, Technische Hochschule Graz, 1971, Fig. 25).

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